

Comment on “Optical orbital angular momentum from the curl of polarization”

Recently, Wang *et al.* reported prediction and observation of the “*new category of optical orbital angular momentum (OAM)*” [1]. It is known that the angular momentum (AM) of light is divided into the spin angular momentum (SAM), associated with the polarization helicity and the OAM, associated with the azimuthal phase gradient [2]. We argue that the AM described in [1] is not a new OAM, but rather represents the *well-known SAM of light*. Moreover, *paraxial theory* used by Wang *et al.* cannot adequately describe their experiment with *tightly focused field*. In our opinion, the orbital motion of the particles observed in [1] is caused by the OAM generated as a result of *spin-to-orbit AM conversion* upon focusing by a high-NA objective [3].

The linear momentum density (or energy flow density) of an optical field is proportional to the time-averaged Poynting vector, which for monochromatic complex electric field $\mathbf{E}(\mathbf{r})e^{-i\omega t}$ reads $\mathbf{P} \propto \text{Im}[\mathbf{E}^* \times (\nabla \times \mathbf{E})]$. In the first paraxial approximation, taking into account small z -components of the field, one has

$$\mathbf{E} = [\mathbf{E}_\perp + ik^{-1}\hat{\mathbf{e}}_z(\nabla \cdot \mathbf{E}_\perp)]e^{ikz}. \quad (1)$$

Here $\mathbf{E}_\perp = A(x, y)[\alpha(x, y)\hat{\mathbf{e}}_x + \beta(x, y)\hat{\mathbf{e}}_y]$ is the transverse electric field, where the complex amplitude $A = ue^{i\psi}$ and normalisation $|\alpha|^2 + |\beta|^2 = 1$ are assumed [1]. It was examined in details recently [4–8] that the momentum density can be divided into orbital and spin parts: $\mathbf{P} = \mathbf{P}^O + \mathbf{P}^S$, which in the paraxial approximation yield

$$\mathbf{P}^O \propto \text{Im}[\mathbf{E}^* \cdot (\nabla) \mathbf{E}], \quad \mathbf{P}^S \propto \nabla \times \text{Im}[\mathbf{E}^* \times \mathbf{E}] / 2. \quad (2)$$

Using Eq. (1) and neglecting corrections from $E_z \propto k^{-1}$, we derive the transverse momentum densities (2):

$$\mathbf{P}_\perp^O \propto u^2 \nabla \psi + u^2 \text{Im}[\alpha^* \nabla \alpha + \beta^* \nabla \beta], \quad (3a)$$

$$\mathbf{P}_\perp^S \propto \nabla \times (u^2 \sigma \hat{\mathbf{e}}_z) / 2, \quad (3b)$$

where $\sigma = 2\text{Im}(\alpha^* \beta)$ is the polarization helicity. The two summands of Eq. (3a) and Eq. (3b) correspond to the terms $\mathbf{P}_\perp^{(1),(2)}$ and $\mathbf{P}_\perp^{(3)}$, Eq. (1), in [1]. According to [2, 4–8] the OAM and SAM densities are given by $\mathbf{L} = \mathbf{r} \times \mathbf{P}^O$ and $\mathbf{S} = \mathbf{r} \times \mathbf{P}^S$, and using Eqs. (3), we obtain

$$L_z \propto u^2 \partial_\phi \psi + u^2 \text{Im}[\alpha^* \partial_\phi \alpha + \beta^* \partial_\phi \beta], \quad (4a)$$

$$S_z \propto r \partial_r (u^2 \sigma) / 2. \quad (4b)$$

Evidently, the terms $J_z^{(1),(2),(3)}$, Eq. (2) in [1], which were interpreted as OAM, correspond to the two terms of the OAM, Eq. (4a), and the SAM, Eq. (4b). Thus, the “new category of the OAM” described by $J_z^{(3)}$ is nothing but the SAM of light. Wang *et al.* considered an example with nearly uniform intensity where the SAM originates exclusively from the helicity gradient $\partial_r \sigma$, but in the general case it arises from both σ and u^2 gradients [4].

In the experiment [1], a paraxial state $\mathbf{E} \simeq \mathbf{E}_\perp$ with nonuniform polarization $\partial_r \sigma \neq 0$ was prepared, which carries finite S_z and $L_z = 0$ (because \mathbf{E}_\perp was ϕ -independent). After that, the field was tightly focused by high-NA objective (NA=0.7), $\mathbf{E} \rightarrow \mathbf{E}^f$, and apparently the AM contributions were calculated from paraxial Eqs. (4) for the resulting field \mathbf{E}_\perp^f (Fig. 3b in [1]). This is erroneous since the focused field is *significantly non-paraxial*, and the longitudinal component $E_z^f \neq 0$ must be taken into account. Apparently, $J_z^{(3)}$ in Fig. 3b [1] is a part of the SAM of the nonparaxial field. At the same time, tight focusing is known [3, 8–10] to produce spin-to-orbit AM conversion. It generates non-zero OAM $L_z^f \propto E_z^{f*} \partial_\phi E_z^f$, because the E_z^f component contains vortex $e^{i\sigma\phi}$ for the circularly polarized ($\sigma = \pm 1$) fields even if \mathbf{E}_\perp was ϕ -independent [3, 8–10]. In the postparaxial approximation this OAM can be estimated as $\sim \theta_0^2/4$ [9], where θ_0 is the aperture angle. Hence, the spin-to-orbit conversion is about 10% for the aperture angles $\theta_0 \sim 30^\circ \div 40^\circ$, which is sufficient to cause the orbital motion of particles observed in [1].

Finally, we remark that mechanical action of both the spin and orbital energy flows on particles crucially depend on the particle properties [5, 8]. Particles used in [1] are rather large compared to the typical scale of the AM density variations, and the assumption of the local action of the momentum density cannot be justified.

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